THE FRANKLIN SQUARES.

THE following letter with magic squares of 8×8 and 16×16 are copied from "Letters and papers on Philosophical subjects by Benjamin Franklin, LL. D., F.R.S.," a work which was printed in London, England, in 1769.

FROM BENJAMIN FRANKLIN ESQ. OF PHILADELPHIA,

TO PETER COLLINSON ESQ. AT LONDON.

DEAR SIR:

According to your request I now send you the arithmetical curiosity of which this is the history.

Being one day in the country at the house of our common friend, the late learned Mr. Logan, he showed me a folio French book filled with magic squares, wrote, if I forget not by one Mr. Frenicle, in which he said the author had discovered great ingenuity and dexterity in the management of numbers; and though several other foreigners had distinguished themselves in the same way, he did not recollect that any one Englishman had done anything of the kind remarkable.

I said it was perhaps a mark of the good sense of our mathematicians that they would not spend their time in things that were merely *difficiles nugæ,* incapable of any useful application. He answered that many of the arithmetical or mathematical questions publicly proposed in England were equally trifling and useless. Perhaps the considering and answering such questions, I replied, may not be altogether useless if it produces by practice an habitual readiness and exactness in mathematical disquisitions, which readiness may, on many occasions be of real use. In the same way says he, may the making of these squares be of use. I then confessed to him that in my younger days, having once some leisure

(which I still think I might have employed more usefully) I had amused myself in making these kind of magic squares, and, at length had acquired such a knack at it, that I could fill the cells of any magic square of reasonable size with a series of numbers as fast as I could write them, disposed in such a manner that the sums of every row, horizontal, perpendicular or diagonal, should be equal; but not being satisfied with these, which I looked on as common and easy things, I had imposed on myself more difficult tasks, and succeeded in making other magic squares with a variety of properties, and much more curious. He then showed me several in the same book of an uncommon and more curious kind; but as I thought none of them equal to some I remembered to have made, he desired me to let him see them; and accordingly the next time I visited him, I carried him a square of 8 which I found among my

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

Fig. 1.

old papers, and which I will now give you with an account of its properties (see Fig. 1). The properties are:

1. That every straight row (horizontal or vertical) of 8 numbers added together, makes 260, and half of each row, half of 260.

2. That the bent row of 8 numbers ascending and descending diagonally, viz., from 16 ascending to 10 and from 23 descending to 17 and every one of its parallel bent rows of 8 numbers make 260, etc., etc. And lastly the four corner numbers with the four middle numbers make 260. So this magical square seems perfect in its kind, but these are not all its properties, there are 5 other curious ones which at some time I will explain to you.

Mr. Logan then showed me an old arithmetical book in quarto,

wrote, I think by one Stifelius, which contained a square of 16 which he said he should imagine to be a work of great labour; but if I forget not, it had only the common properties of making the same sum, viz., 2056 in every row, horizontal, vertical and diagonal.

200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	185
58	39	26	7	250	231	218	199	186	167	154	135	122	103	90	71
198	219	230	251	6	27	38	59	70	91	102	123	134	155	166	187
60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65

Fig. 2.

5	8	9	12
14	15	2	3
11	10	7	6
4	1	16	13

Fig. 3.

Not willing to be outdone by Mr. Stifelius, even in the size of my square, I went home, and made that evening the following magical square of 16 (see Fig. 2) which besides having all the properties of the foregoing square of 8, i. e., it would make 2056 in all the

same rows and diagonals, had this added, that a four-square hole being cut in a piece of paper of such a size as to take in and show through it just 16 of the little squares, when laid on the greater square, the sum of the 16 numbers so appearing through the hole, wherever it was placed on the greater square should likewise make 2056. This I sent to our friend the next morning, who after some days sent it back in a letter with these words:

"I return to thee thy astonishing
"or most stupendous piece
"of the magical square in which"....

—but the compliment is too extravagant and therefore, for his sake, as well as my own I ought not to repeat it. Nor is it necessary,

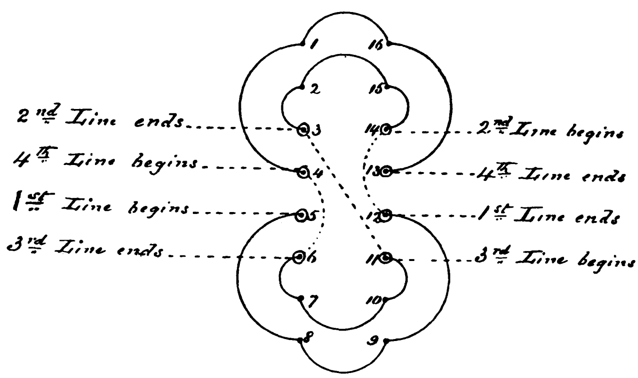

Fig. 4.

for I make no question but you will readily allow the square of 16 to be the most magically magical of any magic square ever made by any magician.

I am etc.

B. F.

It will be seen that the squares shown in Figs. 1 and 2 are not perfect according to the rules of even magic squares previously given, but the interesting feature of their *bent diagonal columns* calls for more than passing notice. In order to facilitate the study of their construction, a 4×4 square is given in Fig. 3 which presents similar characteristics.

The dotted lines in this square indicate four bent diag-

onal columns, each of which has a total of 34; three of these columns being intact within the square and one being broken. Four bent diagonal columns may be formed from each of the four sides of the square, but only twelve

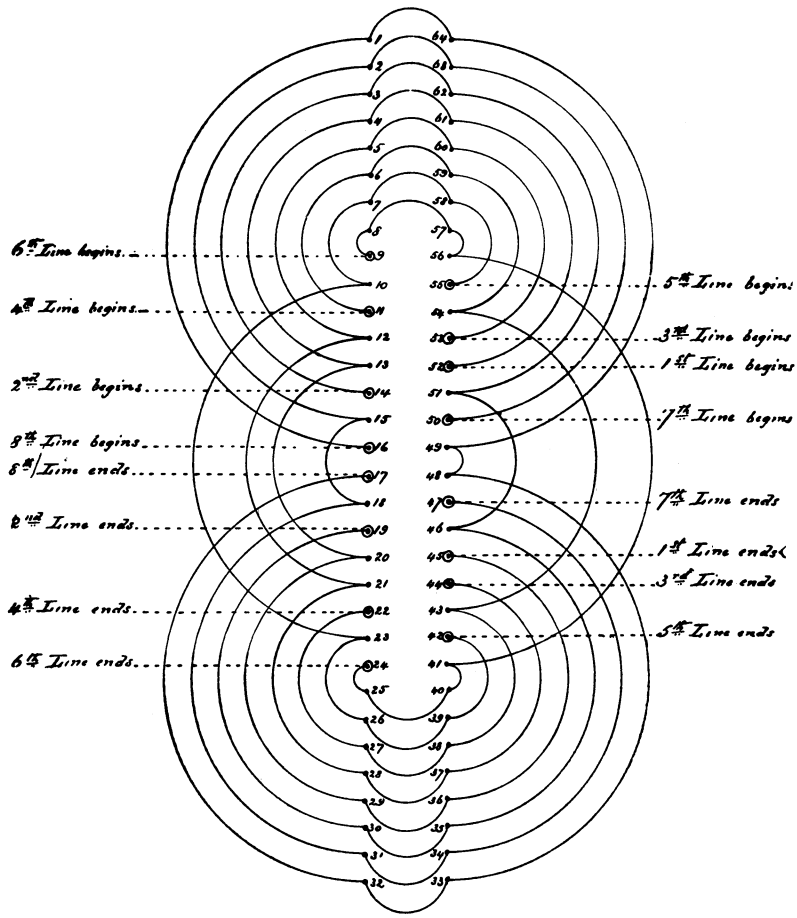

Fig. 5.

of these sixteen columns have the proper totals. Adding to these the eight straight columns, we find that this square contains twenty columns with summations of 34. The 4×4 "Jaina" square contains sixteen columns which sum

up to 34 while the ordinary 4×4 magic square contains only ten.

The 8×8 Franklin square (Fig. 1) contains forty-eight columns which sum up to 260, viz., eight horizontal, eight perpendicular, sixteen bent horizontal diagonals, and sixteen bent perpendicular diagonals, whereas the ordinary 8×8 magic square contains only eighteen columns of the same summation.

In addition to the other characteristics mentioned by Franklin in his letter concerning his 8×8 magic square

5	8	57	60
54	55	10	11
43	42	23	22
28	25	40	37

Section 1. (Top.)

59	58	7	6
12	9	56	53
21	24	41	44
38	39	26	27

Section 2.

62	63	2	3
13	16	49	52
20	17	48	45
35	34	31	30

Section 3.

4	1	64	61
51	50	15	14
46	47	18	19
29	32	33	36

Section 4. (Bottom.)

Fig. 6.

it may be stated that the sum of the numbers in any 2×2 sub-square contained therein is 130, and that the sum of any four numbers that are arranged symmetrically equidistant from the center of the square also equals 130.

In regard to his 16×16 square, Franklin states in his letter that the sum of the numbers in any 4×4 sub-square contained therein is 2056. The sub-division may indeed be carried still further, for it will be observed that the sum of the numbers in any 2×2 sub-square is 514, and there are also other curious features which a little study will disclose.

The Franklin Squares possess a unique and peculiar

symmetry in the arrangement of their numbers which is not clearly observable on their faces, but which is brought out very strikingly in their geometrical diagrams as given

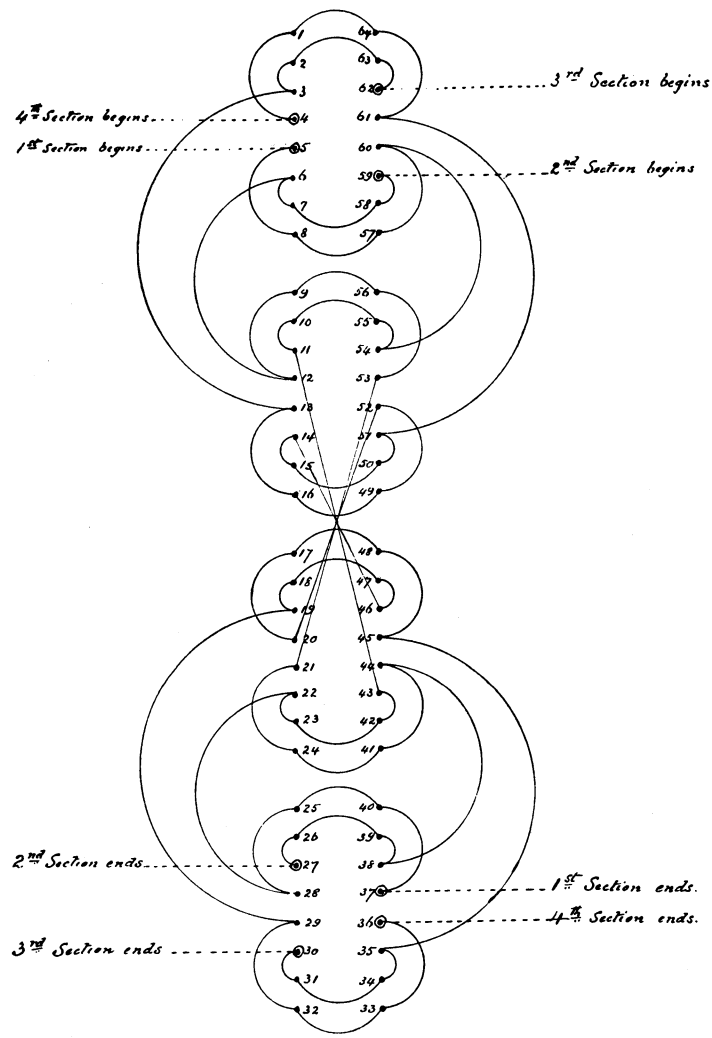

Fig. 7.

in Figs. 4 and 5, which illustrate respectively the diagrams of the 4×4 and 8×8 squares.

Magic cubes may be readily constructed by expanding these diagrams and writing in the appropriate numbers.

The cube of $4 \times 4 \times 4$ and its diagram are given as examples in Figs. 6 and 7, and it will be observed that the curious characteristics of the square are carried into the cube.

W. S. ANDREWS.

SCHENECTADY, May, 28, 1906.